Teaching and Learning for Transference

The question whether algebra deserves its prominent role in the high school curriculum was raised once again on July 28 by Andrew Hacker in a New York Times opinion column, “Is Algebra Necessary?” [1]. His piece echoes the argument made a year ago by Sol Garfunkel and David Mumford in the opinion pages of the New York Times, “How to Fix our Math Education” [2]. It also resonates with Mike Shaughnessy's comments in his President's Column for the NCTM newsletter, “Endless Algebra—the Deadly Pathway from High School Mathematics to College Mathematics” [3].

There have been many thoughtful responses to the Hacker article. For a clear explanation why algebra is important, the best is still Zal Usiskin’s piece from 1995, “Why is Algebra Important to Learn?” [4]. Daniel Willingham posted a good reply to Hacker on his blog, “Yes, algebra is necessary” [5]. In a private communication [6], Dan Kennedy of the Baylor School in Chattanooga describes the beauty of algebra and laments the fact that we are doing such a very poor job of communicating that beauty.

I want to focus this column on a theme raised in Lynn Steen’s response, “Reflections on Mathematics and Democracy” [7]. His article was conceived as a reaction to the Garfunkel and Mumford editorial, but he also discusses Hacker. As Steen points out, Hacker’s argument is not that algebra is not important; it is that algebra is not working in the curriculum. The problem—as Steen distills it—is the fact that many if not most students never learn how to transfer the knowledge and skills that are taught in algebra. They are trapped within a perception of algebra as a system of arcane manipulations with no relevance to anything outside the mathematics classroom.

Steen’s solution focuses on the curriculum: embedding applications into mathematics courses, team-teaching cross-disciplinary courses, and employing project- or career-focused curricula. I fully agree that the curriculum can be and has been an obstacle to learning how to transfer one’s knowledge and skills. I also recognize that the problem is greater than just the curriculum.

Teaching for transference is one of the most difficult tasks we face as educators. I would like to share two personal stories that serve as touchstones for me.

In my last year at Penn State, 1993–94, I taught a yearlong honors course in calculus using the Project CALC materials developed by David Smith and Lang Moore. CALC stands for Calculus As a Laboratory Course. The course met five days a week, two of those days in computer labs where the students explored and applied the ideas of calculus. The classroom was used both to prepare students for the laboratories and to reflect on and distill what had happened there. Some students put up initial resistance to such an unconventional approach, and a few switched to a regular section at winter break, but most came to enjoy learning this way. They believed they were getting much more than they would have from traditional instruction.

Midway through spring semester, one of my students came breathlessly to my office. She had just completed an engineering exam where, as she told me, she had forgotten the formula needed to solve one of the problems. But then she remembered what she had learned in my course, and she figured out how to solve the problem with what she knew from calculus.

I do not know what the problem was or what tools she used, but she made it clear that she was relying on the ideas, on the conceptual knowledge she had acquired. I knew then that my job with her was done. Whatever specific information she might still learn from me, nothing would equal the power that came from recognizing that she did not have to rely on memorized procedures, that she was capable of applying the principles of calculus to derive solutions to problems that mattered to her. Not that she always would do this, but now she knew that she could.

The second story is about myself. During the spring of 7th grade, I met periodically after school with my math teacher, Mr. Checkley. He presented and challenged me with bits of mathematics. One of these consisted of the rules for determining divisibility by 3, 9, or 11 by considering the digits of the integer. He asked me to find an explanation why these rules work. I struggled, convincing myself that they are always valid but unable to frame a proof.

The following year I took Algebra I. I had some difficulty at first with this strange kind of mathematics, always valid but unable to frame a proof.
as everyone did, until I realized that what I was learning in this class was exactly the language that I needed to provide Mr. Checkley with his proof. Once I knew that algebra is simply a language for exploring and explaining mathematical patterns, a language that I could use to answer mathematical questions of interest to me, it became easy.

There are several lessons that I take from these experiences. First, transference need not be to a real-world application. Nor is it about the need to use one’s mathematical knowledge in a career. It is important because of the power that comes from discovering that one can rely on one’s own reasoning to recover a forgotten formula or uncover the logic behind an unexpected pattern.

Second, I have learned that what works in a particular setting with one instructor and a particular group of students will not necessarily work when these parameters are changed. My Penn State class consisted of University Scholars, selected from the top 3% of the student body. I was highly motivated to make it work. Project Calc was used at Duke with mixed results. No curriculum by itself will ever be sufficient.

Third and finally, learning for transference is a process that takes time. A student begins with new knowledge, either acquired through personal discovery or introduced and explained by a teacher. This is followed by an opportunity to apply this knowledge in a fresh context. The next steps are critical and far too often neglected. The student must now reflect on what worked and what difficulties were encountered, then distill the significant features that made it work. Even now the process is not done. There must be a fresh attempt at application, followed once again by reflection and distillation. I have found that most students need to progress through this cycle several times before they have real control of a new piece of knowledge. I struggle with the difficulties of engaging my students in this process while balancing the demands of the course. A continuing challenge for me is to decide which concepts deserve this much attention.

I work with privileged and highly motivated students. The difficulties inherent in accomplishing learning for transference in our public schools are far greater, but the goal should be the same. Lynn Steen has asked us to “organize[e] the curriculum to pay greater attention to the goal of transferable knowledge and skills.” I would go beyond this. We need to organize the very way we teach so that we keep this end in mind.


